Flywheel Shot Analysis: Correction and Extension

FRC Team 449 - The Blair Robot Project
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1 Introduction

One of the most common calculations in FRC design, flywheel shooter calculators help designers inform their flywheel shooter parameters by estimating the projectile exit velocity using values such as the flywheel speed, size, and inertia. Most flywheel calculators are sourced from FRC Team 846's 2012 "Shooter Calculations Document," which models a single-flywheel hooded shooter with a no-slip assumption using conservation of angular momentum.

In July 2023, FRC Team 199's mentor Dean Brettle published "Flywheel Shooter Analysis," proposing a different approach to the problem. To summarize, Team 846's conservation of angular momentum approach models a closed system where the flywheel starts with a certain amount of angular momentum, which gets shared with the ball to launch it using an inelastic collision model. Dean, however, points out that this approach fails to consider the force applied by the hood (which is outside the system) onto the ball, which would change the total amount of momentum in the flywheel-ball system. On top of that, the conservation of angular momentum formulation was incorrectly set up.

When starting this research, I was looking for a flywheel model that a) includes projectile initial velocity to model the effects of the feeder system on shot consistency, and b) includes a hood roller opposite the main flywheel with some fixed gear ratio between the two. Dean Brettle also highlighted these as areas of future investigation.

In this paper, I will derive a new model to predict the projectile exit velocity for a dual-flywheel or hood-roller flywheel, where the two opposite flywheels are mechanically coupled with a known fixed speed ratio between them. In Appendix A, I will separately derive an expanded single-flywheel hooded shooter model that includes the initial velocity of the projectile which aligns with both the dual-flywheel model and the prior solution by Dean Brettle. I will then compare this corrected model to ReCalc's calculator in Appendix B.

Note that in this paper, the terms "shooter wheel" and "flywheel" may be used interchangeably and refer to the system of spinning shooter wheels, inertia wheels, shafts, gears, etc as a whole.

The latest version of this whitepaper is available here. A spreadsheet including the derived calculations is available here. If either of these links break, please check the original ChiefDelphi post or contact me (@rafi) on ChiefDelphi.

2 Double Flywheel or Hood Roller Flywheel Shooter

2.1 Problem Statement

We model the double-flywheel hooded shooter with these assumptions and definitions:

1. At the end of the shot, there is no slip between the ball and either flywheel.
2. The effect of compression is negligible.
3. Both flywheel centers of mass align with their respective axes of rotation.
4. The ball and both flywheels are perfectly rigid bodies.
5. The ball has radius r_b, mass m_b, moment of inertia I_b about its center of mass, initial angular velocity ω_{bi}, and initial linear velocity V_{bi} perpendicular to the radius to the flywheel center.
6. The first flywheel has radius r_{w1} and initial angular velocity ω_{w1i}.
7. The second flywheel has radius r_{w2} and initial angular velocity ω_{w2i}
8. The second flywheel spins at a rate $\omega_{w2} = G\omega_{w1}$, where if $G > 1$ the second flywheel spins faster than the first.
9. The flywheel system's total virtual moment of inertia at the first flywheel's axis, including both flywheel sets, is I_w.

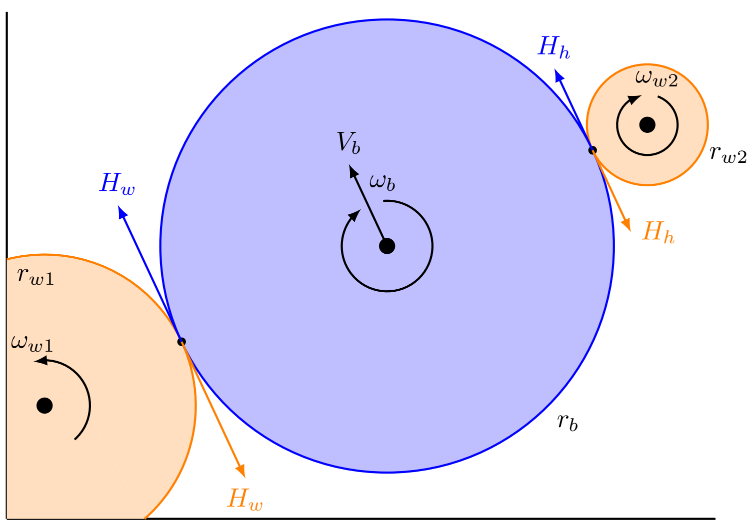
We want to find the projectile exit velocities V_{bf} and ω_{bf}, and the flywheel final velocities ω_{w1f} and ω_{w2f}.

Note that this analysis works regardless of which flywheel is the "first" flywheel.

2.2 Initial Setup

The problem geometry and direction definitions are shown below. Note that the hood roller impulse is shown as pushing the ball forwards, but this may not necessarily be true. If the hood roller impulse pushes the ball backwards instead, we should get a negative value for H_h.

Conversely, if the hood roller spins really fast, we may end up with a negative value for ω_b, where the ball has top spin instead of backspin.



2.3 Solution

We start with the non-slip condition and rigid body equations. We know that at the point of contact between the first flywheel and the ball there is no slip, so:

$$\omega_{w1} r_{w1} = V_b + r_b \omega_b$$

We know the same is true for the second flywheel, so:

$$\omega_{w2} r_{w2} = V_b - r_b \omega_b$$

Adding the two equations:

$$\omega_{w1} r_{w1} + \omega_{w2} r_{w2} = 2V_b$$

$$V_b = \frac{\omega_{w1} r_{w1} + \omega_{w2} r_{w2}}{2}$$

$$V_b = \frac{\omega_{w1}(r_{w1} + G r_{w2})}{2} \tag{1}$$

If we instead subtract the two equations:

$$\omega_{w1} r_{w1} - \omega_{w2} r_{w2} = 2 r_b \omega_b$$

$$r_b \omega_b = \frac{\omega_{w1} r_{w1} - \omega_{w2} r_{w2}}{2}$$

$$r_b \omega_b = \frac{\omega_{w1}(r_{w1} - G r_{w2})}{2} \tag{2}$$

Summing the two linear impulses will give us the change in linear momentum of the ball:

$$m_b(V_{bf} - V_{bi}) = H_w + H_h \tag{3}$$

And we can find the change in angular momentum of the ball as:

$$I_b(\omega_{bf} - \omega_{bi}) = H_w r_b - H_h r_b$$

$$\frac{I_b}{r_b}(\omega_{bf} - \omega_{bi}) = H_w - H_h \tag{4}$$

To find the impulses' impact on the flywheel system, we first examine the impulse at the hood roller.

We know the angular impulse applied onto the top roller about its center is:

$$J_{w2,w2} = H_h r_{w2}$$

$$\int \tau_{w2} dt = H_h r_{w2}$$

Since the hood roller spins G times faster, in reverse, due to a gear ratio to the main roller, any torque applied onto the hood roller can instead be thought of as being applied $-G$ times more to the main roller:

$$\int \tau_{w2 \to w1} dt = -G H_h r_{w2}$$

$$J_{w2,w1} = -G H_h r_{w2}$$

The impulse applied directly to the first wheel by its contact with the ball is:

$$J_{w1,w1} = -H_w r_{w1}$$

So the total impulse applied to the flywheel system, when analyzed from the main flywheel's point of view, is:

$$J_{w1} = -H_w r_{w1} - G H_h r_{w2}$$

So:

$$I_w \omega_{w1f} - I_w \omega_{w1i} = -H_w r_{w1} - G H_h r_{w2} \tag{5}$$

We now have five unknowns and five equations. We start by rewriting 5:

$$-(I_w \omega_{w1f} - I_w \omega_{w1i}) = r_{w1} H_w + G r_{w2} H_h$$

$$\begin{aligned}-(I_w \omega_{w1f} - I_w \omega_{w1i}) = \frac{1}{2}(&r_{w1} H_w + r_{w1} H_w + G r_{w2} H_h + G r_{w2} H_h \\ &+ r_{w1} H_h - r_{w1} H_h + G r_{w2} H_w - G r_{w2} H_w)\end{aligned}$$

$$\begin{aligned}I_w \omega_{w1i} - I_w \omega_{w1f} = \frac{1}{2}[&(r_{w1} H_w + r_{w1} H_h + G r_{w2} H_w + G r_{w2} H_h) \\ &+ (r_{w1} H_w - r_{w1} H_h - G r_{w2} H_w + G r_{w2} H_h)]\end{aligned}$$

$$I_w \omega_{w1i} - I_w \omega_{w1f} = \frac{1}{2}[(r_{w1} + G r_{w2})(H_w + H_h) + (r_{w1} - G r_{w2})(H_w - H_h)]$$

$$I_w \omega_{w1i} - I_w \omega_{w1f} = \frac{r_{w1} + G r_{w2}}{2}(H_w + H_h) + \frac{r_{w1} - G r_{w2}}{2}(H_w - H_h)$$

Plugging in equations 3 and 4:

$$I_w \omega_{w1i} - I_w \omega_{w1f} = \frac{r_{w1} + G r_{w2}}{2} m_b (V_{bf} - V_{bi}) + \frac{r_{w1} - G r_{w2}}{2} \frac{I_b}{r_b} (\omega_{bf} - \omega_{bi})$$

$$\begin{aligned}I_w \omega_{w1i} - I_w \omega_{w1f} = &\frac{r_{w1} + G r_{w2}}{2} m_b V_{bf} + \frac{r_{w1} - G r_{w2}}{2} \frac{I_b}{r_b} \omega_{bf} \\ &- \frac{r_{w1} + G r_{w2}}{2} m_b V_{bi} - \frac{r_{w1} - G r_{w2}}{2} \frac{I_b}{r_b} \omega_{bi}\end{aligned}$$

$$\begin{aligned}&I_w \omega_{w1i} + \frac{r_{w1} + G r_{w2}}{2} m_b V_{bi} + \frac{r_{w1} - G r_{w2}}{2} \frac{I_b}{r_b} \omega_{bi} = \\ &\qquad \frac{r_{w1} + G r_{w2}}{2} m_b V_{bf} + \frac{r_{w1} - G r_{w2}}{2} \frac{I_b}{r_b} \omega_{bf} + I_w \omega_{w1f}\end{aligned}$$

Plugging in equations 1 and 2:

$$\begin{aligned}&I_w \omega_{w1i} + \frac{r_{w1} + G r_{w2}}{2} m_b V_{bi} + \frac{r_{w1} - G r_{w2}}{2} \frac{I_b}{r_b} \omega_{bi} = \\ &\qquad \frac{r_{w1} + G r_{w2}}{2} m_b \frac{r_{w1} + G r_{w2}}{2} \omega_{w1f} + \frac{r_{w1} - G r_{w2}}{2} \frac{I_b}{r_b} \frac{r_{w1} - G r_{w2}}{2 r_b} \omega_{w1f} + I_w \omega_{w1f}\end{aligned}$$

$$... = m_b \left(\frac{r_{w1} + Gr_{w2}}{2}\right)^2 \omega_{w1f} + I_b \left(\frac{r_{w1} - Gr_{w2}}{2r_b}\right)^2 \omega_{w1f} + I_w \omega_{w1f}$$

$$... = \left(m_b \left(\frac{r_{w1} + Gr_{w2}}{2}\right)^2 + I_b \left(\frac{r_{w1} - Gr_{w2}}{2r_b}\right)^2 + I_w\right) \omega_{w1f}$$

$$I_w \omega_{w1i} + \frac{r_{w1} + Gr_{w2}}{2} m_b V_{bi} + \frac{r_{w1} - Gr_{w2}}{2} \frac{I_b}{r_b} \omega_{bi} = \frac{1}{4}\left(m_b \left(r_{w1} + Gr_{w2}\right)^2 + \frac{I_b}{r_b^2}\left(r_{w1} - Gr_{w2}\right)^2 + 4I_w\right) \omega_{w1f}$$

$$\omega_{w1f} = \frac{4I_w \omega_{w1i} + 2(r_{w1} + Gr_{w2}) m_b V_{bi} + 2(r_{w1} - Gr_{w2}) \frac{I_b}{r_b} \omega_{bi}}{m_b \left(r_{w1} + Gr_{w2}\right)^2 + \frac{I_b}{r_b^2}\left(r_{w1} - Gr_{w2}\right)^2 + 4I_w}$$

$$\omega_{w1f} = \frac{4I_w \omega_{w1i} + 2(r_{w1} - Gr_{w2}) \frac{I_b}{r_b} \omega_{bi} + 2(r_{w1} + Gr_{w2}) m_b V_{bi}}{4I_w + \left(r_{w1} - Gr_{w2}\right)^2 \frac{I_b}{r_b^2} + \left(r_{w1} + Gr_{w2}\right)^2 m_b}$$

As an aside, when we plug in $G = 0$, we get a result matching the single-flywheel hooded shooter formula from Appendix A

$$\omega_{w1f} = \frac{4I_w \omega_{w1i} + 2r_{w1} \frac{I_b}{r_b} \omega_{bi} + 2r_{w1} m_b V_{bi}}{4I_w + r_{w1}^2 \frac{I_b}{r_b^2} + r_{w1}^2 m_b}$$

Hood Roller Flywheel Equations

$$\omega_{w1f} = \frac{4I_w \omega_{w1i} + 2(r_{w1} - Gr_{w2}) \frac{I_b}{r_b} \omega_{bi} + 2(r_{w1} + Gr_{w2}) m_b V_{bi}}{4I_w + \left(r_{w1} - Gr_{w2}\right)^2 \frac{I_b}{r_b^2} + \left(r_{w1} + Gr_{w2}\right)^2 m_b}$$

$$V_{bf} = \frac{r_{w1} + Gr_{w2}}{2} \omega_{w1f}$$

$$\omega_{bf} = \frac{r_{w1} - Gr_{w2}}{2r_b} \omega_{w1f}$$

$$\omega_{w2f} = G \omega_{w1f}$$

2.4 Compound Flywheel Shooter

While some double flywheel shooters are fed directly, many hooded shooters with hood rollers are actually composed of two parts - one section with a static hood backing followed by a section with powered hood rollers. These can be thought of as a single-flywheel hooded shooter feeding into a double-flywheel shooter. We can use the equations we've found to solve for the behavior of this two-stage compound flywheel shooter. Unfortunately, the math doesn't work out to give a clean formula for the compound flywheel shooter in one step, so it is recommended to solve for the system final condition after the single-flywheel portion and pass that in as the initial condition of the double-flywheel portion.

2.5 Efficiency

Interestingly, this two-step construction allows the flywheel system to exceed Dean Brettle's 50% max efficiency finding for a single-flywheel shooter, which we find also holds for a standalone double-flywheel shooter. The maximum achievable efficiencies are listed in the table below. These efficiencies are generally achieved by having a high flywheel moment of inertia and by selecting specific ratios between the flywheels. Total efficiency is the proportion of lost flywheel kinetic energy that is transformed into the ball's linear and rotational kinetic energy, while linear efficiency is only the proportion that's transformed into linear kinetic energy. The three listed projectile types are a hollow cylinder where all projectile mass is concentrated on the outside for maximum moment of inertia (worst case), a solid uniformly dense sphere, and a projectile where all the mass is concentrated in the center for minimum moment of inertia (best case).

Max Shooter Efficiencies			
	Single	Double	Compound
Hollow Cylinder	Linear: 25.0% Total: 50.0%	50.0%	50.0%
Solid Sphere	Linear: 35.71% Total: 50.0%	50.0%	58.8%
Dense Core	50.0%	50.0%	66.7%
For Double and Compound, both efficiencies match when $r_{w1} = Gr_{w2}$. Both efficiencies match for all Dense Core cases. Compound efficiencies are maximized in Dense Core when $r_{w1} = Gr_{w2}$.			

3 Conclusion

The theoretical equations for single and double flywheel shooters are presented. A spreadsheet including the derived calculations is available here.

Expanding on Dean Brettle's findings regarding shooter efficiency, both single and double flywheel shooters have a max kinetic energy transfer efficiency of 50% starting from rest, but a double flywheel shooter can allocate more of that kinetic energy into linear motion, which is generally desirable. Additionally, passing the ball into the shooter with initial velocity doesn't significantly increase the exit velocity, but drastically increases the efficiency of the shot, particularly for double flywheel shooters. Modeling hood-roller flywheel shooters as a single-flywheel shooter followed by a double-flywheel shooter allows the system to exceed the 50% efficiency limit.

The updated single-flywheel model also significantly differs from ReCalc's flywheel calculator due to the erroneous assumptions that fed into the original derivation. In particular, it predicts slightly higher flywheel exit velocity and drastically higher flywheel energy drain, which directly drives design decisions like flywheel inertia, motor count, and gearing.

Next steps for investigation could include validating these theoretical models against empirical test data, as the assumptions that feed into these models are unverified. In particular, whether the "no slip at exit" condition is realistic has not been empirically validated, and some other assumptions (e.g. rigid body and negligible compression assumptions), which aren't true in reality, may have a significant impact on results.

I'd like to give major thanks to Dean Brettle from FRC Team 199 for his initial findings and his review of this whitepaper.

Appendix A Single-Flywheel Hooded Shooter

A.1 Problem Statement

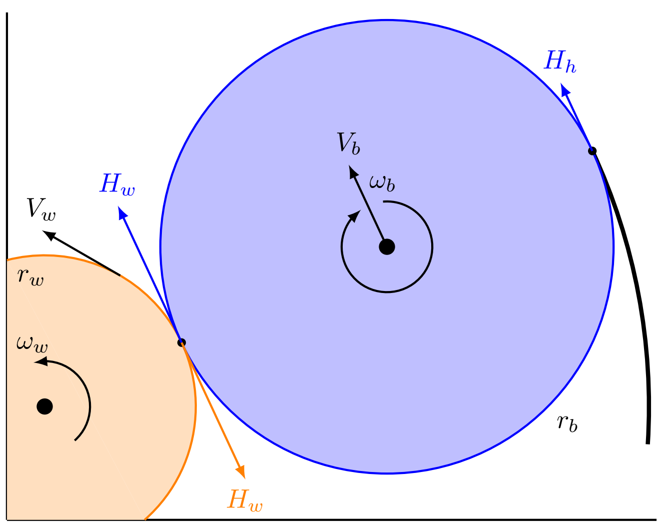
We model the single-flywheel hooded shooter with these assumptions and definitions:

1. At the end of the shot, there is no slip between the ball and the flywheel or between the ball and the hood.
2. The effect of compression is negligible.
3. The flywheel center of mass aligns with its axis of rotation.
4. The ball, flywheel, and hood are perfect rigid bodies.
5. The ball has radius r_b, mass m_b, moment of inertia I_b about its center of mass, initial angular velocity ω_{bi}, and initial linear velocity V_{bi} perpendicular to the radius to the flywheel center.
6. The flywheel has radius r_w, moment of inertia I_w around its center of mass and initial angular velocity ω_{wi}.

We want to find the projectile exit velocities V_{bf} and ω_{bf}, and the flywheel final velocity ω_{wf}.

Note that any additional inertia wheels' inertias should be included in I_w. E.g. if the shooter wheel has a moment of inertia I_s and an inertia wheel with moment of inertia I_f at a 2.5:1 speed-increasing gear ratio, we would have $I_w = I_s + 2.5^2 I_f$

A.2 Initial Setup



Let H_w be the total linear impulse transferred between the flywheel and the ball, and H_h be the total linear impulse transferred between the ball and the hood. Perhaps counterintuitively, in order to limit the ball's rotation from slipping out, the hood pushes the ball forwards instead of pulling it back, so H_h points in the direction of the shot.

The ball's total angular momentum $L_{b,w}$ about the flywheel center is:

$$L_{b,w} = m_b V_b (r_w + r_b) - I_b \omega_b$$

And the change in the ball's total angular momentum is:

$$L_{b,wf} - L_{b,wi} = H_w r_w + H_h(r_w + 2r_b)$$

So:

$$(m_b V_{bf}(r_w + r_b) - I_b\omega_{bf}) - (m_b V_{bi}(r_w + r_b) - I_b\omega_{bi}) = H_w r_w + H_h(r_w + 2r_b) \quad (6)$$

A.3 Solution

First, we establish some basic relationships. When the ball exits the shooter, we know that its velocity at the flywheel point of contact is V_w, and its velocity at the hood point of contact is 0.

$$V_w = V_b + \omega_b r_b$$
$$0 = V_b - \omega_b r_b$$

From the second equation:

$$V_b = \omega_b r_b \quad (7)$$

And adding the two equations:

$$V_w + 0 = V_b + \omega_b r_b + V_b - \omega_b r_b$$
$$V_w = 2V_b \quad (8)$$

Next we'll find some equations relating the two impulses and some of the velocities. The ball's total angular momentum $L_{b,b}$ about its own center is:

$$L_{b,b} = -I_b\omega_b$$

Note that $L_{b,b}$ has a negative term as ω_b is defined to be CW-positive while the flywheel is defined to be CCW-positive. The two impulses applied to the ball change its angular momentum by:

$$L_{b,bf} = L_{b,bi} - H_w r_b + H_h r_b$$
$$-I_b\omega_{bf} = -I_b\omega_{bi} - H_w r_b + H_h r_b$$
$$\omega_{bf} = \omega_{bi} + \frac{r_b}{I_b}(H_w - H_h) \quad (9)$$

Next, the flywheel's total angular momentum $L_{w,w}$ about its own center is:

$$L_{w,w} = I_w\omega_w$$

The impulse between the flywheel and the ball changes its angular momentum by:

$$L_{w,wf} = L_{w,wi} - H_w r_w$$
$$I_w\omega_{wf} = I_w\omega_{wi} - H_w r_w$$
$$\omega_{wf} = \omega_{wi} - \frac{r_w}{I_w}H_w \quad (10)$$

We perform some algebraic manipulation on the right hand side of equation 6 with the goal of applying equation 9:

$$... = H_w r_w - [(H_w - H_h)(r_w + 2r_b) - H_w(r_w + 2r_b)]$$

$$... = H_w r_w - \left[\frac{I_b r_b}{I_b r_b}(H_w - H_h)(r_w + 2r_b) - H_w(r_w + 2r_b)\right]$$

$$... = H_w r_w - \left[\frac{I_b}{r_b}\frac{r_b}{I_b}(H_w - H_h)(r_w + 2r_b) - H_w(r_w + 2r_b)\right]$$

We now plug in 9 to eliminate the H_h term:

$$... = H_w r_w - \left[\frac{I_b}{r_b}(\omega_{bf} - \omega_{bi})(r_w + 2r_b) - H_w(r_w + 2r_b)\right]$$

$$... = H_w r_w + H_w(r_w + 2r_b) - \frac{I_b(r_w + 2r_b)}{r_b}(\omega_{bf} - \omega_{bi})$$

$$... = H_w(2r_w + 2r_b) - \frac{I_b(r_w + 2r_b)}{r_b}(\omega_{bf} - \omega_{bi})$$

We continue with algebraic manipulation, this time to apply equation 10:

$$... = \frac{-I_w r_w}{-I_w r_w} H_w(2r_w + 2r_b) - \frac{I_b(r_w + 2r_b)}{r_b}(\omega_{bf} - \omega_{bi})$$

$$... = -\frac{I_w(2r_w + 2r_b)}{r_w}\left(-\frac{r_w}{I_w}H_w\right) - \frac{I_b(r_w + 2r_b)}{r_b}(\omega_{bf} - \omega_{bi})$$

$$... = -\frac{I_w(2r_w + 2r_b)}{r_w}\left(\omega_{wi} - \frac{r_w}{I_w}H_w\right) + \frac{I_w(2r_w + 2r_b)}{r_w}\omega_{wi} - \frac{I_b(r_w + 2r_b)}{r_b}(\omega_{bf} - \omega_{bi})$$

And we plug in equation 10:

$$... = -\frac{I_w(2r_w + 2r_b)}{r_w}\omega_{wf} + \frac{I_w(2r_w + 2r_b)}{r_w}\omega_{wi} - \frac{I_b(r_w + 2r_b)}{r_b}(\omega_{bf} - \omega_{bi})$$

$$... = -\frac{I_w(2r_w + 2r_b)}{r_w}(\omega_{wf} - \omega_{wi}) - \frac{I_b(r_w + 2r_b)}{r_b}(\omega_{bf} - \omega_{bi})$$

Writing the full equation back out, we want to express all final velocity unknowns in terms of V_{bf} using equations 7 and 8:

$$(m_b V_{bf}(r_w + r_b) - I_b\omega_{bf}) - (m_b V_{bi}(r_w + r_b) - I_b\omega_{bi}) = \\ -\frac{I_w(2r_w + 2r_b)}{r_w}(\omega_{wf} - \omega_{wi}) - \frac{I_b(r_w + 2r_b)}{r_b}(\omega_{bf} - \omega_{bi})$$

$$\left(m_b V_{bf}(r_w + r_b) - I_b\frac{V_{bf}}{r_b}\right) - (m_b V_{bi}(r_w + r_b) - I_b\omega_{bi}) = \\ -\frac{I_w(2r_w + 2r_b)}{r_w}\left(\frac{V_{wf}}{r_w} - \omega_{wi}\right) - \frac{I_b(r_w + 2r_b)}{r_b}\left(\frac{V_{bf}}{r_b} - \omega_{bi}\right)$$

$$\left(m_b(r_w+r_b)-\frac{I_b}{r_b}\right)V_{bf}-(m_bV_{bi}(r_w+r_b)-I_b\omega_{bi})=$$
$$-\frac{2I_w(2r_w+2r_b)}{r_w^2}V_{bf}-\frac{I_b(r_w+2r_b)}{r_b^2}V_{bf}+\frac{I_w(2r_w+2r_b)}{r_w}\omega_{wi}+\frac{I_b(r_w+2r_b)}{r_b}\omega_{bi}$$

$$\left(m_b(r_w+r_b)-\frac{I_b}{r_b}\right)V_{bf}+\frac{2I_w(2r_w+2r_b)}{r_w^2}V_{bf}+\frac{I_b(r_w+2r_b)}{r_b^2}V_{bf}=$$
$$\frac{I_w(2r_w+2r_b)}{r_w}\omega_{wi}+\frac{I_b(r_w+2r_b)}{r_b}\omega_{bi}+(m_bV_{bi}(r_w+r_b)-I_b\omega_{bi})$$

$$\left(m_b(r_w+r_b)+\frac{2I_w(2r_w+2r_b)}{r_w^2}+\frac{I_b(r_w+2r_b)}{r_b^2}-\frac{I_b}{r_b}\right)V_{bf}=$$
$$\frac{I_w(2r_w+2r_b)}{r_w}\omega_{wi}+\left(\frac{I_b(r_w+2r_b)}{r_b}-I_b\right)\omega_{bi}+m_bV_{bi}(r_w+r_b)$$

$$\left(m_b(r_w+r_b)+\frac{2I_w(2r_w+2r_b)}{r_w^2}+\frac{I_b(r_w+r_b)}{r_b^2}\right)V_{bf}=$$
$$\frac{I_w(2r_w+2r_b)}{r_w}\omega_{wi}+\frac{I_b(r_w+r_b)}{r_b}\omega_{bi}+m_b(r_w+r_b)V_{bi}$$

$$V_{bf}=\frac{\frac{2I_w(r_w+r_b)}{r_w}\omega_{wi}+\frac{I_b(r_w+r_b)}{r_b}\omega_{bi}+m_b(r_w+r_b)V_{bi}}{\frac{4I_w(r_w+r_b)}{r_w^2}+\frac{I_b(r_w+r_b)}{r_b^2}+m_b(r_w+r_b)}$$

Hooded Flywheel Equations

$$\omega_{wf}=\frac{\frac{4I_w}{r_w}\omega_{wi}+\frac{2I_b}{r_b}\omega_{bi}+2m_bV_{bi}}{r_w\left(\frac{4I_w}{r_w^2}+\frac{I_b}{r_b^2}+m_b\right)}=\frac{4I_wr_b^2\omega_{wi}+2I_br_wr_b\omega_{bi}+2m_br_wr_b^2V_{bi}}{4I_wr_b^2+I_br_w^2+m_br_w^2r_b^2}$$

$$V_{bf}=\frac{\frac{2I_w}{r_w}\omega_{wi}+\frac{I_b}{r_b}\omega_{bi}+m_bV_{bi}}{\frac{4I_w}{r_w^2}+\frac{I_b}{r_b^2}+m_b}=\frac{2I_wr_wr_b^2\omega_{wi}+I_br_w^2r_b\omega_{bi}+m_br_w^2r_b^2V_{bi}}{4I_wr_b^2+I_br_w^2+m_br_w^2r_b^2}$$

$$\omega_{bf}=\frac{\frac{2I_w}{r_w}\omega_{wi}+\frac{I_b}{r_b}\omega_{bi}+m_bV_{bi}}{r_b\left(\frac{4I_w}{r_w^2}+\frac{I_b}{r_b^2}+m_b\right)}=\frac{2I_wr_wr_b\omega_{wi}+I_br_w^2\omega_{bi}+m_br_w^2r_bV_{bi}}{4I_wr_b^2+I_br_w^2+m_br_w^2r_b^2}$$

To verify the flywheel-centered angular momentum impulse approach used to establish equation 6, we can derive the change in the ball's linear momentum using the two impulses we solved for above:

From equation 10:

$$2H_w=-\frac{2I_w}{r_w}(\omega_{wf}-\omega_{wi})$$

From equation 9:

$$H_w - H_h = \frac{I_b}{r_b}(\omega_{bf} - \omega_{bi})$$

Subtract these two equations and simplify:

$$H_w + H_h = -\frac{2I_w}{r_w}(\omega_{wf} - \omega_{wi}) - \frac{I_b}{r_b}(\omega_{bf} - \omega_{bi})$$

Expressing all the final velocities in terms of V_{bf} using equations 7, 8, and the V_{bf} equation in the red box:

$$H_w + H_h = -\frac{2I_w}{r_w}\left(\frac{2V_{bf}}{r_w} - \left[\left(\frac{4I_w}{r_w^2} + \frac{I_b}{r_b^2} + m_b\right)V_{bf} - \frac{I_b}{r_b}\omega_{bi} - m_b V_{bi}\right] / \frac{2I_w}{r_w}\right) - \frac{I_b}{r_b}\left(\frac{V_{bf}}{r_b} - \omega_{bi}\right)$$

$$H_w + H_h = -\frac{4I_w V_{bf}}{r_w^2} + \left[\left(\frac{4I_w}{r_w^2} + \frac{I_b}{r_b^2} + m_b\right)V_{bf} - \frac{I_b}{r_b}\omega_{bi} - m_b V_{bi}\right] - \frac{I_b V_{bf}}{r_b^2} + \frac{I_b}{r_b}\omega_{bi}$$

$$H_w + H_h = m_b V_{bf} - m_b V_{bi}$$

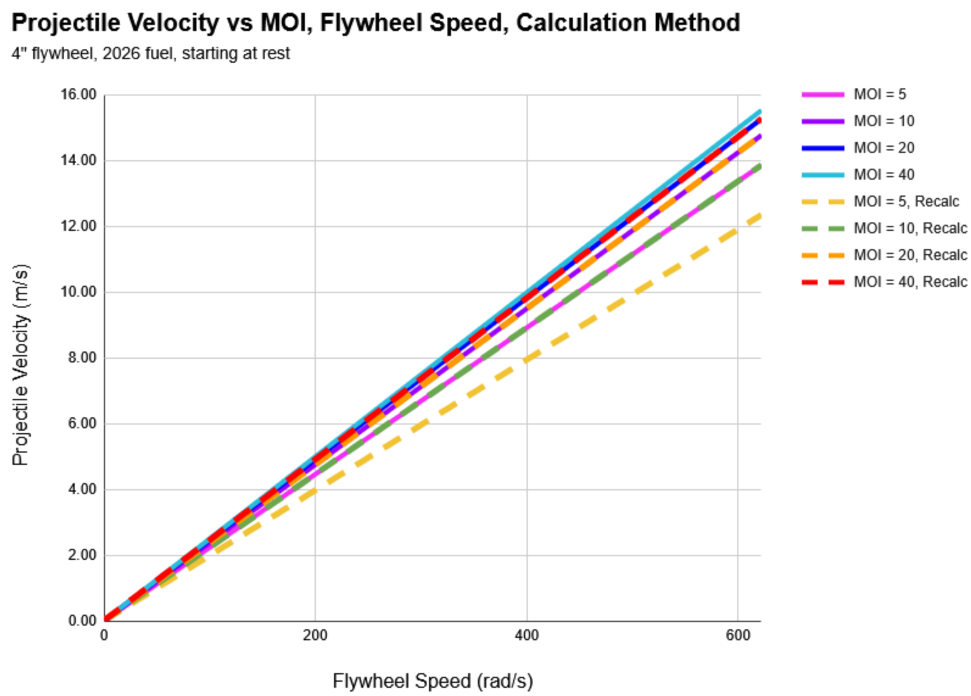
$$H_w + H_h = m_b \Delta V_b$$

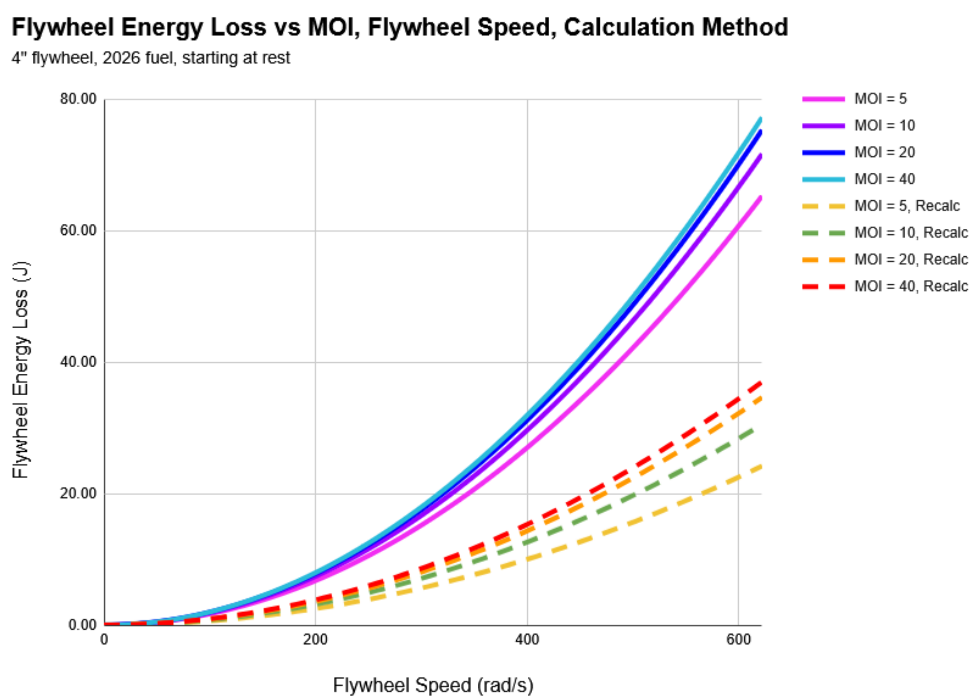
This shows that using our solution, the total linear impulse applied to the ball equals its change in linear momentum.

Appendix B Comparison with ReCalc

In the companion spreadsheet there's a sheet for comparing the results of the single-flywheel hooded shooter to what ReCalc, the current standard flywheel calculator, predicts. The two plots below quantify the impact of the calculation errors for an FRC 2026 context, not including the other additional capabilities of the new model such as initial velocity.

When calculating the exit velocity, ReCalc's calculations effectively cut the flywheel inertia in half, resulting in a lower exit velocity. Additionally, ReCalc treats the energy transfer as 100% efficient (when under the corrected model it's at most 50% efficient), which drastically underestimates the amount of energy depleted from the flywheel. This results in an underestimate of the post-shot recovery time, which may impact design decisions that teams made including inertia wheel sizing, motor count, and motor gearing.


Projectile Velocity vs MOI, Flywheel Speed, Calculation Method
4" flywheel, 2026 fuel, starting at rest
Projectile Velocity (m/s)
Flywheel Speed (rad/s)
0.00
2.00
4.00
6.00
8.00
10.00
12.00
14.00
16.00
0
200
400
600
MOI = 5
MOI = 10
MOI = 20
MOI = 40
MOI = 5, Recalc
MOI = 10, Recalc
MOI = 20, Recalc
MOI = 40, Recalc


Flywheel Energy Loss vs MOI, Flywheel Speed, Calculation Method
4" flywheel, 2026 fuel, starting at rest
Flywheel Energy Loss (J)
Flywheel Speed (rad/s)
0.00
20.00
40.00
60.00
80.00
0
200
400
600
MOI = 5
MOI = 10
MOI = 20
MOI = 40
MOI = 5, Recalc
MOI = 10, Recalc
MOI = 20, Recalc
MOI = 40, Recalc